Real and Complex Dynamical Systems

International conference on the occasion of
Prof. Yulij Ilyashenko's 75th birthday

Moscow, November 26–30, 2018

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Talks

Irreducible representations of finitely generated nilpotent groups

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At ICM 2010 Parshin conjectured that irreducible complex representations of finitely generated nilpotent groups are monomial if and only if they have finite weight. Every irreducible complex representation is known to be monomial for finite nilpotent groups and for unitary irreducible representations of connected nilpotent Lie groups (due to A.A. Kirillov and J. Dixmier). We prove Parshin's conjecture in full generality. We will also discuss how to approach the moduli space problem for such representations, and its relation to higher-dimensional adeles. For nilpotency classes 2 and 3 we obtained a full description of the coarse moduli space.

Geometry of Foliations over Number Fields and Applications

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Many problems related to the study of limit cycles and the Hilbert 16th problem are equivalent to counting the roots for certain types of (univariate) analytic functions. One approach to such zero-counting problems is to compare the number of zeros of an analytic function with its asymptotic growth using the Jensen formula. Yulij Ilyashenko has been a champion of this approach, applying it together with his students to several classes of functions including solutions of linear analytic ODEs, abelian integrals and Poincaré maps of planar systems.

In this talk I will discuss recent progress on higher dimensional analogs of the growth-vs-zeros method, i.e. studying solutions of systems of equations in several variables by looking at their asymptotic growth. More specifically, I will show how this technique leads to good bounds when the equations consist of functions satisfying overdetermined systems of algebraic PDEs over number fields. This class notably contains many functions of importance in arithmetic geometry including elliptic, abelian, modular and Riemann theta functions. I will also sketch without much technical detail how applying this technique to such functions one can obtain effective estimates for various problems in diophantine geometry (e.g. Manin-Mumford, André-Oort, Masser-Zannier and Katz-Oort).

Positive Lyapunov exponents for quasi-periodically driven circle endomorphisms with critical points

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We consider skew-products over irrational rotation on the circle, where the fiber maps are circle endomorphisms which have a unique critical point. For a class of such maps (whose critical points are "non-flat") we show that the (fibered) Lyapunov exponents are positive for a.e. initial condition.

This is a joint work with H. Eliasson.

Heat kernel and counting problems for Kleinian groups.

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Given a group acting discretely and properly on a metric space, we are interested in estimating the number of points belonging to both the orbit of a given point and to a ball of fixed center when the radius of the ball goes to infinity. Any control of this asymptotic is a counting theorem.

In this talk we will focus on Kleinian groups, i.e groups acting on a hyperbolic space. We will start with the historical background, and proceed with the proposal of a strategy to obtain counting theorems based on the use of the heat kernel.

On phase-lock areas in model of Josephson effect and double confluent Heun equations

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In 1973 B. Josephson received Nobel Prize for discovering a new fundamental effect in superconductivity concerning a system of two superconductors separated by a very narrow dielectric: there could exist a supercurrent tunneling through this junction (called *Josephson junction*). One of the known models of the overdamped Josephson junction used a family of non-linear first order differential equations. Motivated by problems on Josephson effect, V. M. Buchstaber, O. V. Karpov and S. I. Tertychnyi studied this family in a series of papers (see JETP 2001 and later). They introduced and studied an equivalent family of dynamical systems on 2-torus. Motivated by dynamical systems point of view, Yu. S. Ilyashenko and J. Guckenheimer suggested and studied a subfamily of this family on 2-torus as an interesting slow-fast system in 2001. Analogous systems arose in different models of mathematics, mechanics and physics that a priori were not related one to the other.

The classical rotation number of the family of dynamical systems on 2-torus is a function of parameters of the family (identified with the average voltage through a long time interval). The r-th level set of the rotation number function

is called the *r-th phase-lock area,* if it has non-empty interior. Our family is very atypical: the phase-lock areas exist only for integer values of the rotation number, in difference to the classical Arnold tongue picture (the quantization effect discovered by V. M. Buchstaber, O. V. Karpov and S. I. Tertychnyi in 2010). The complexification of our family is the projectivization of a family of classical double confluent Heun equations: second order linear differential equations on the Riemann sphere with two irregular singular points at 0 and infinity.

In the talk we will present results on geometric and analytic properties of the boundaries of the phase-lock areas. We will discuss several important geometric conjectures on the phase-lock areas and analytic conjectures on Heun equations and relations between them. We will present the state of art in this area including recent works of V. M. Buchstaber, S. I. Tertychnyi, A. A. Glutsyuk, V. A. Kleptsyn, D. A. Filimonov, I. V. Schurov.

Acknowledgements. Research of V. M. Buchstaber was supported by Russian Foundation for Basic Research (RFBR) grants 14-01-00506, 17-01-00192. Research of A. A. Glutsyuk was supported by Russian Science Foundation (RSF) grant 18-41-05003, by Russian Foundation for Basic Research (RFBR) grants 13-01-00969-a, 16-01-00748, 16-01-00766, and by French National Research Agency (ANR) grant ANR-13-JS01-0010.

Classification of surfaces transverse to flows: the case of the geodesic flow

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Given a non singular vector field, transverse hypersurfaces (when they exist) are good tools to study the flow dynamics. The goal of this talk is, in dimension 3 and for the particular case of the geodesic flow on a hyperbolic surface, to classify such surfaces. For this, we introduce an elementary tool, the intersection norms, associated to a finite collection of closed curves on a surface. These norms are simple cousins of the Thurston-Gromov norms and, like their cousins, their dual unit balls are integer polytopes. The integer points in these polytopes yield the classification we are looking for.

Structural stability in the moduli space of algebraic foliations of the complex projective plane

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I will prove the existence of a non trivial stability component in the space of degree two algebraic foliations of the complex projective plane, consisting of foliations having a non trivial Fatou component: a fibration by discs over a Riemann surface of genus three. This result can be opposed to the famous result of Yu. Ilyashenko (adressed at the ICM 78) establishing

minimality, ergodicity and rigidity for generic foliations of the plane that preserve a line, and its generalizations by F. Loray/J. Rebelo or T. Golenishcheva-Kutusova/V. Kleptsyn.

Sums of Cantor sets in spectral theory

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Structure of sums of Cantor sets is a classical question related to many problems in dynamical systems, number theory, and geometric measure theory. We will provide an overview of currently known results and open questions, and discuss numerous recent applications to spectral theory of Schrödinger operators (the results of joint projects with D. Damanik, J. Fillman, and B. Solomyak).

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On hyperbolic attractors and topology of ambient manifolds

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According to S. Smale, Ya. Sinai, J. Franks, S. Newhouse and E. Manning, a manifold M^n which admits codimension one Anosov diffeomorphism is the torus. A similar result is obtained By V. Z. Grines and E. V. Zhuzhoma in the case when the non-wandering set of structurally stable diffeomorphism contains orientable expanding attractor of codimension one (see [1] for information and references). In [2] the topological classification of three-dimensional manifolds have been obtained for manifolds admitting structurally stable diffeomorphisms whose nonwandering set consists of two-dimensional attractors and repellers. Moreover it has been obtained complete topological classification of described above classes of diffeomorphisms.

The report is supported by the HSE Basic Research Program (project no. 95) in 2018.

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On the multipliers at fixed points of rational self-maps of the projective plane

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For a rational self-map of the complex projective space of dimension n, $f : \mathbf{P}^n \to \mathbf{P}^n$, we may consider the collection of the eigenvalues of its derivative at its fixed points. It depends only on the linear equivalence class of f. These numbers cannot be arbitrary. For instance, the holomorphic Lefschetz fixed-point theorem gives an algebraic relation between them, and more of these can be obtained through the Atiyah-Bott fixed-point theorem. However, a parameter-counting argument shows that there are algebraic relations among these multipliers that are not obtained through these theorems. What do we know of these relations? Can we give them explicitly? To what extent does the collection of the eigenvalues at the fixed points determine the linear equivalence class of a rational self-map? We do not have answers to these questions even in the simplest case, the one where $n = 2$ and where f is given by quadratic polynomials.

We will discuss these problems for those rational maps of the projective plane which are given by quadratic polynomials and which have, moreover, a forward-invariant line. It is a joint work with Valente Ramírez (Rennes).

Surface groups of germs of analytic diffeomorphisms in one variable

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Let $\mathrm{Diff}(\mathbf{C}, 0)$ be the group of germs of analytic diffeomorphisms at the origin $0 \in \mathbf{C}$, i.e. of power series in one variable with positive radius of convergence and with non-zero derivative at 0. We prove that this group contains subgroups isomorphic to the fundamental group of any closed orientable surface. This is a joint work with Cantat, Cerveau and Souto.

Normal distribution of correlation measures of binary sum-of-digits functions

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I will explain a joint result with Jordan Emme. We study correlation measures for the sum of digits in base 2. Denote by $\mu_a(d)$ the asymptotic density of the set $\mathcal{E}_{a,d} = \{n \in \mathbb{N},\ s_2(n+a) - s_2(n) = d\}$ (where s_2 is the sum-of-digits function in base 2). Then, for any point X in $\{0,1\}^{\mathbb{N}}$, define the integer sequence $(a_X(n))_{n\in\mathbb{N}}$ such that the binary decomposition of $a_X(n)$ is the prefix of length n of X. We prove that for *any* shift-invariant ergodic probability measure ν on $\{0,1\}^{\mathbb{N}}$, the sequence $\left(\mu_{a_X(n)}\right)_{n\in\mathbb{N}}$ satisfies a central limit theorem.

A generalization of the diffeomorphism by Smale with hyperbolic attractor

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Smale in 1967 [1] proposed a method of constructing diffeomorphisms with hyperbolic attractors using a expanding endomorphism. This construct was implemented explicitly using expanding endomorphism of the circle so that the result is a three-dimensional diffeomorphisms of the torus with solenoidal one-dimensional expanding attractor which is locally homeomorphic to the product of the cut and Cantor set. We offer a generalization of this construction, where instead of expanding endomorphism of the circle is regarded As the endomorphism of the (multidimensional) torus [2, 3]. Examines the type of transitive invariant sets of the corresponding diffeomorphisms.

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Schrödinger operators with quasi-periodic potentials and the Aubry-Mather theory

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Schrödinger operators with quasi-periodic potentials were intensively studied in the last few decades. Their spectral properties depend on the value of the coefficient in front of the potential, so called. coupling constant. For small values of the coupling constant the spectrum is absolutely continuous, while for large coupling constants the spectrum is pure point. Natural families of Schrödinger operators with quasi-periodic potentials appear in the context of the Aubry-Mather theory. In this setting there are no coipling constants. Instead operators depend on the nonlinearity parameter for related area-preserving maps. We shall discuss the transition from the absolutely continuous to the pure point spectrum for such families of Schrödinger operators.

Furstenberg theorem: now with a parameter!

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Consider a random product of i.i.d. matrices, randomly chosen from $\mathrm{SL}(2,\mathbb{R})$:

$$T_n = A_n ... A_2 A_1,$$

where the random matrices A_i are i.i.d..

A classical Furstenberg theorem then implies, that under some very mild nondegeneracy conditions (no finite common invariant set of lines, no common invariant metric) for the law of A_i's the norm of such a product almost surely grows exponentially.

Now, what happens if each of these matrices $A_i(s)$ depends on an additional parameter s, and hence so does their product $T_n(s)$? For each individual s, the Furstenberg theorem is still applicable. However, what can be said almost surely for the random products $T_n(s)$, depending on a parameter?

We will impose a few reasonable additional assumptions, of which the most important is that the dependence of angle is monotonous w.r.t. the parameter: increasing the parameter "rotates all the directions clockwise".

It turns out that, under these assumptions,

- Almost surely for all the parameter values, except for a zero Hausdorff dimension (random) set, the Lyapunov exponent exists and equals to the Furstenberg one.
- Almost surely for all the parameter values the upper Lyapunov exponent equals to the Furstenberg one

- At the same time, in the no-uniform-hyperbolicity parameter region there exists a dense subset of parameters, for each of which the lower Lyapunov exponent takes any fixed value between 0 and the Furstenberg exponent.

Our results are related to the Anderson localization in dimension one, providing a purely dynamical viewpoint on its proof.

This is a joint work [1] with Anton Gorodetski.

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Convergence of spherical averages for actions of Fuchsian Groups

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The talk is based on the joint work [1] with A. Bufetov and C. Series.

Consider an action of a Fuchsian group G on a Lebesgue probability space (X, μ) by measure-preserving maps T_g. Let $\mathcal{R}$ be a fundamental domain of G and let G_0 be defined as above. Assume that $\mathcal{R}$ has *even angles*, that is, the boundary of the tesselation $\{g\mathcal{R}\}$ is a union of complete geodesic

lines. Finally, for $\varphi \in L^1(X,\mu)$ define its spherical averages as follows:

$$\mathbf{S}_n(\varphi) = \frac{1}{\#\{g : |g| = n\}} \sum_{g:|g|=n} \varphi \circ T_g,$$

where $|g|$ is a norm of $g \in G$ with respect to the generating set G_0.

Theorem 1. *Let G be a non-elementary Fuchsian group G and suppose it has a fundamental domain $\mathcal{R}$ with even corners and satisfying a technical condition (in particular, it holds for any domain with at least 5 sides). Let G_0 be a symmetric set of transformations defined above.*

Let G act on a Lebesgue probability space (X,μ) by measure-preserving transformations. Denote by $\mathcal{I}_{G_0^2}$ the σ-algebra of sets invariant under all maps $T_{g_1 g_2}$, $g_1, g_2 \in G_0$. Then, for any function $f \in L\log L(X,\mu)$, the sequence $(\mathbf{S}_{2n}(f))$ converges as $n \to \infty$ almost surely and in L^1 to the conditional expectation $\mathbb{E}(f|\mathcal{I}_{G_0^2})$ with respect to the σ-algebra $\mathcal{I}_{G_0^2}$.

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Ergodicity, mixing and KAM

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To prove the ergodicity for Hamiltonian systems of big or infinite dimension is a notoriously complicated problem of high importance. But what we often have in physics are not Hamiltonian systems, but systems of the form <Hamiltonian system> + <small dissipation> + <small random forcing>, where the random forcing may be very degenerate (i.e. it affects only a few modes). For such systems an analogy of the ergodicity is called the mixing. In my talk I will remind the definition of the mixing and explain how the KAM-theory provides a powerful tool to prove the mixing for the systems above. The talk is based on a joint work with Armen Shirikyan and Vahagn Nersesyan [1]

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On skew products over a quasi-periodic shift of the circle

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As is known, skew products over irrational shift on the circle arise as Poincaré mappings for quasi-periodic nonautonomous systems with two incommensurable frequencies. Many results for such skew products exists when the related bundle over circle is linear (reducibility, localization, spectral properties, etc.). Much work was done for the structure of 1-dimensional skew products (with 1-dimensional leaves): non-smooth bifurcations, rotation sets, and so forth. In this talk I would like to touch the topic of structural dynamics for such skew products, including classification problems.

The first topic is the dynamics of skew products with the circle as a leaf. Here there is a simple way to distinguish vertically hyperbolic diffeomorpisms. For such diffeomorphisms the classifying theorem is valid: the rotation number for a shift, the integer and some permutation is the complete set of invariants. Examples are presented when absence of vertical hyperbolicity leads to a dimension inhomogeneity of the invariant sets. Analogs of such skew products in greater dimensions will be discussed (analogs of gradient-like skew products, Morse-Smale products).

The situation is much more involved when the leaves are n-dimensional submanifolds ($n \geq 2$) and intersection of nonwandering set with leaves are infinite. Then even in the case of vertical hyperbolicity the dynamics can be very complicated. Related examples will be discussed.

Lagrange and Markov are very different from Hausdorff's point of view

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The Lagrange and Markov spectra are subsets of $\mathbb{R}$ arising naturally in the study of certain Diophantine approximation problems, heights of cusp excursions of geodesics of the modular surface, etc.

After reviewing several features of these spectra, we will discuss our joint work with C. G. Moreira on the difference between them: in particular, we will describe our solution to a conjecture of Cusick from 1975.

Construction of formulas for the coefficients of the asymptotic expansion of the correspondence map and the monodromy transformation

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An algorithm for distinguishing between stable and unstable focus has long been known by calculating the coefficients of the monodromy transform series in the case when the singular point has a Newton diagram consisting of one edge. In the case when the Newton diagram of a vector field consists of several edges, the monodromy transformation can be decomposed into a composition of so-called extended correspondence

mappings, each of which corresponds to the inner vertex of the Newton diagram. An algorithm for constructing formulas for the coefficients of the asymptotic decomposition (of arbitrary length) of the extended correspondence mapping is developed in the absence of some resonances of the saddle obtained as a result of a blow-up with a center at this internal vertex. The power of a set of exceptional resonances depends on the length of the asymptotic expansion. The constructed algorithm can be implemented on a computer.

Partially supported by RFBR (project 17-01-00739-a)

Integrable magnetic geodesic flows on 2-torus: new example via quasi-linear system of PDEs

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The only one example has been known of magnetic geodesic flow on the 2-torus which has a polynomial in momenta integral independent of the Hamiltonian. In this example the integral is linear in momenta. In the talk the problem of integrability on one energy level will be considered. This problem can be reduced to a remarkable Semi-hamiltonian system of quasi-linear PDEs and to the question of existence of smooth periodic solutions for this system. Our main result states that the pair of Liouville metric with zero magnetic field on the 2-torus can be analytically deformed to a Riemannian metric with small magnetic field so that the magnetic geodesic flow on an energy level is integrable by means of a quadratic in momenta integral. The result were obtained with Michael Bialy (Tel Aviv) and Sergey Agapov (Novosibirsk).

Complex Cellular Structures

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Real semialgebraic sets admit so-called cellular decomposition, i.e. representation as a union of cells diffeomorphic to cubes, with some very convenient properties. Attempts to build a straightforward complex holomorphic generalization of this construction meet difficulties related to inner metric properties of holomorphic sets, absent in real case.

I will explain these difficulties and propose a proper complex generalization. The construction is motivated by and relates to uniformization results of resolution of singularities theory, by "growth-zeros" ideology used in Infinitesimal Hilbert 16th problem and by Yomdin—Gromov parameterization results. As a corollary, we solve an old Yomdin's conjecture about tail entropy and prove new results in transcendental number theory.

https://arxiv.org/abs/1802.07577

A C^1 Anosov diffeomorphism with a horseshoe that attracts almost any point

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Joint work with C. Bonatti, S. Minkov, and I. Shilin.

Using Bowen's construction of a horseshoe with positive Lebesgue measure ([1]), we construct an example of a C^1 Anosov diffeomorphism of the 2-torus that admits a physical measure ν such that

- the basin of ν has full Lebesgue measure,
- ν is supported on a horseshoe $\mathcal{H}$ with zero Lebesgue measure.

The horseshoe $\mathcal{H}$ is *semithick*, i.e. it is a product of a Cantor set with positive measure in the unstable direction and a Cantor set with zero measure in the stable direction.

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Analytic invariants of germs of curves and foliations in $(\mathbb{C}^2, 0)$

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This talk deals with two geometric objects: on one side, the germs of analytic curves in $(\mathbb{C}^2, 0)$ and their analytic moduli, and on the other, the analytic functional invariants of germs of holomorphic dicritic foliations in $(\mathbb{C}^2, 0)$. We prove that any analytic class of germs of singular curves in $(\mathbb{C}^2, 0)$ having $n+1$ pairwise transversal smooth branches can be realized in an appropriate dicritic foliation with prescribed collection of involutions. This collection of involutions is related to the tangency points with the exceptional divisor of the corresponding dicritic foliation; we recall that the involutions (with fixed parametrization) of a generic dicritic foliation is one of the, so called, Thom's invariants (the functional one) of analytic classification of generic dicritic foliations.

The purpose of this talk is to talk about a recent work with Jessica Jaurez [1], and to give as well an overview of results previously obtained in joint works with Ernesto Rosales and Sergei Voronin [2, 3]. All together they constitute some steps towards the thorough understanding of the geometry of the finite parametric (analytic) invariants of singular curves and singular foliations (work in progress).

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Negative refraction tiling billiards

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A straight line moving through a standard square tiling is associated to a trajectory of a billiard in a square by an unfolding procedure. The idea of tiling billiards is to work directly with the trajectories moving through the tilings. This talk will be devoted to negative refraction tiling billiards, in which a piecewise linear trajectory breaks each time it crosses a boundary of a tile in a way that the refraction coefficient is equal to -1. It happens that even in a very simple periodic tiling by the same triangle tile, the behaviour of such a system is quiet intricate (even though this is a system of zero entropy): some trajectories of negative refraction tiling billiards approximate fractals (such as Rauzy fractal). I will describe the state of progress, as well as the key ideas behind the understanding of these billiards and related foliations, following the work of P. Baird-Smith, D. Davis, E. Fromm, S. Iyer, P. Hubert and myself.

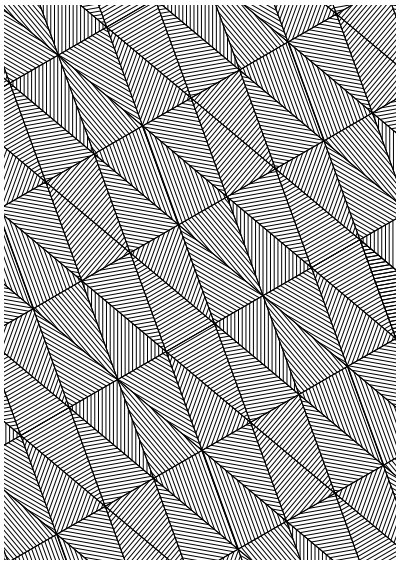

Figure 1: Tiling billiard

On Palis Problem of Embedding of Morse-Smale Cascades into Flows

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The problem of finding of conditions when Morse-Smale cascade can be embedded into a topological flow goes back to J. Palis (1969). He stated some necessary conditions of embedding of Morse-Smale cascade $f : M^n \to M^n$ $(n \geq 2)$ into a topological flow and proved that these conditions are sufficient

for embedding of Morse-Smal diffeomorphism into a flow for $n = 2$. We show that already in the case $n = 3$ they are not sufficient.

The report is devoted to finding a criteria for embedding of Morse-Smale diffeomorphism of the sphere S^n, $n \geq 3$, into topological flow. The results was obtained in collaboration with V. Grines, E. Gurevich and V. Medvedev [1, 2, 3].

Acknowledgements. This work was supported by the Russian Science Foundation (project 17-11-01041).

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Dmitry Andreevich Gudkov – an outstanding mathematician and the researcher of Lobachevsky's biography (to the 100 anniversary since birth)

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Life and activity of professor of the Nizhny Novgorod University Dmitry Andreevich Gudkov (1918 – 1992) are described. The main mathematical result of D.A. Gudkov (1969) – the topological classification of real nonsingular curves of degree 6 – belong to the known for the difficulty 16th Hilbert problem. At the same time Gudkov formulated a conjecture ("Gudkov's congruence") about topological characteristics of nonsingular curve of any even degree. V. I. Arnold's (1971) and V. A. Rokhlin's (1972) works on the proof of this congruence opened the modern stage in development of a subject.

In 1992 D. A. Gudkov published the book "N. I. Lobachevsky. Biography riddles" in which on the basis of archival documents proved that the Nizhny Novgorod land surveyor S. S. Shebarshin was the father of Nikolay Lobachevsky and his brothers. Now this statement turned into the admitted fact for experts.

Patterson–Sullivan construction for point processes and the reconstruction for harmonic functions

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In a recent joint work with Alexander Bufetov, we show that the classical Patterson-Sullivan construction can be generalized to the random setting in the theory of point processes. This construction allows us to recover the value of any harmonic function with additional regularity at any point of the disc from its restriction to a random configuration of the determinant point process with the Bergman kernel. Similar result is then extended to real and complex hyperbolic spaces of higher dimension. Recovering continuous functions by the Patterson-Sullivan construction is also shown to be possible in more general Gromov hyperbolic spaces.

Global approximation on $\mathrm{Diff}^\omega(S^1)$

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We will consider finitely generated subgroups G of the group of (orientation-preserving) analytic diffeomorphisms of the circle, $\mathrm{Diff}^\omega(S^1)$. Among finitely generated subgroups of $\mathrm{Diff}^\omega(S^1)$, groups that are *locally non-discrete* are extremely common. Here a group G as above is said to be *locally non-discrete* if there is a non-trivial sequence of elements $\{g_i\} \subset G$ whose restrictions to some fixed, open (non-empty) interval $I \subset S^1$ converge to the identity in a suitable topology.

Another interesting class of finitely generated subgroups of $\mathrm{Diff}^\omega(S^1)$ is formed by those groups G that are *globally non-discrete* in the sense that G contains a non-trivial sequence of elements g_i converging to the identity on all of S^1. This latter class of groups is also very broad, in particular it contains open sets of groups. This class also contains some natural examples of subgroups of $\mathrm{Diff}^\omega(S^1)$.

In this talk we will discuss denseness properties for both locally non-discrete and globally non-discrete groups G as above. After a quick review of "local approximation" results and their dynamical applications, we will discuss the "global approximation phenomenon". This discussion will focus on a recent approximation result along with some dynamical applications that cannot be derived from the previously known "local approximation" theorems. A typical example in this direction being provided by the problem of existence of regular stationary measures. If time permits, I will also comment on some prob-

lems concerning the borderline between locally non-discrete groups and globally non-discrete ones.

On reduction of singularities for vector fields in dimension 3

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A classical result due to Seidenberg states that every singular holomorphic foliation on a complex surface can be turned into a foliation possessing only *elementary singular points* by means of a finite sequence of (one-point) blow-ups. Here we remind the reader that a singular point p is said to be *elementary* if the foliation $\mathcal{F}$ in question has at least one eigenvalue different from zero at p. However, in dimension 3, the natural analogue of Seidenberg theorem no longer holds as shown by Sanz and Sancho.

More recently, two major works have been devoted to this problem. In [1], Cano, Roche and Spivakovsky have worked out a reduction procedure using (standard) blow-ups. The main disadvantage of their theorem lies, however, in the fact that some of their final models are quadratic and hence have all eigenvalues equal to zero. On the other hand, McQuillan and Panazzolo [2] have successfully used *weighted blow-ups* to obtain a satisfactory desingularization theorem in the category of stacks, rather than in usual complex manifolds.

A basic question is how far these theorems can be improved if we start with a complete vector field on a complex manifold

of dimension 3, rather than with a general 1-dimensional holomorphic foliation. In this context of complete vector fields, we will prove a sharp desingularization theorem. Our proof of the mentioned result will naturally require us to revisit the works of Cano-Roche-Spivakovsky and of McQuillan-Panazzolo on general 1-dimensional foliations. In particular, by building on [1], our discussion will also shed some new light on the desingularization problem for general 1-dimensional foliation on complex manifolds of dimension 3.

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On the classification of germs of foliations in $(C^n, 0)$

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We consider germs of holomorphic vector fields in $(C^n, 0), n \geq 3$, with non isolated singularities. We assume that the set of singular points forms a submanifold of codimension 2, and the sum of the nonzero eigenvalues of the linearization of the germs

at each singular point is zero. We give the orbital analytic classification of generic germs of such type. It happens that the formal classification is trivial, and the analytic one has functional moduli, unlike the cases of dicritic and nondicritic generic germs in n=2 with isolated and degenerated singularity where the formal and analytical classification coincide [2],[3]. Joint work with Ortiz-Bobadilla, L. (UNAM, México); Voronin, S. M. (CSU, Russia).

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Erratic behavior for one-dimensional random walks in a generic quasi-periodic environment

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Consider a Markov chain on a one-dimensional torus $\mathbb{T}$, where a moving point jumps from x to $x \pm \alpha$ with probabilities $p(x)$ and $1 - p(x)$, respectively, for some fixed function $p \in C^\infty(\mathbb{T}, (0,1))$ and $\alpha \in \mathbb{R} \setminus \mathbb{Q}$. Such Markov chains are called random walks in a quasi-periodic environment. It was shown by Ya. Sinai that for Diophantine α the corresponding random walk has an absolutely continuous invariant measure, and the distribution of any point after n steps converges to this measure. Moreover, the Central Limit Theorem with linear drift and variance holds.

In contrast to these results, we show that random walks with a Liouvillian frequency α generically exhibit an erratic statistical behavior. In particular, for a generic p, the corresponding random walk does not have an absolutely continuous invariant measure, both drift and variance exhibit wild oscillations (being logarithmic at some times and almost linear at other times), Central Limit Theorem does not hold.

These results are obtained in a joint work with Dmitry Dolgopyat and Bassam Fayad.

Sparkling saddle loops of vector fields on surfaces

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A vector field on a two-dimensional surface different from the sphere and projective plane can have a dissipative saddle with a saddle loop that attracts the "free" unstable separatrix of this saddle. When the loop is unfolded in a generic one-parameter family, new saddle loops appear for parameter values arbitrarily close to zero. These are sparkling saddle connections which are loops – hence the name.

When we study this as a semi-local bifurcation, it turns out that in the orientable case the closure of the set of parameters for which the field has a saddle loop is a Cantor set. By modifying the field outside a neighborhood of the unstable manifold of our saddle, it is possible to add a fixed but arbitrary number of bifurcation points into each interval of the complement of the Cantor set, thus obtaining countably many nonequivalent bifurcation diagrams for local one-parameter families of vector fields on any surface other than the sphere, Klein bottle, and projective plane.

Cohomological equations for linear involutions

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Famous Roth theorem about diophantine approximations states that a given algebraic number may not have too many rational number approximations, that are "very good". More precisely, Roth first defined a class of numbers that are not very easy to approximate by rationals (they are called Roth numbers) and then showed that almost all algebraic irrationals are of Roth type, and that they form a set of a full measure which is invariant under the natural action of the modular group SL(2,Z).

In addition to their interesting arithmetical properties, Roth type irrationals appear in a study of the cohomological equation associated with a rotation $R_a : R_a(x) = x + a$ of the circle $\mathbb{T} = \mathbb{R}/\mathbb{Z}$: a is of Roth type if and only if for all $r, s : r > s + 1 > 1$ and for all functions Φ of class C^r on $\mathbb{T}$ with zero mean there exists a unique function $\Psi \in C^s(\mathbb{T})$ with zero mean such that

$$\Psi - \Psi \circ R_a = \Phi.$$

In 2005 Marmi, Moussa and Yoccoz established an analogue of Roth theorem for interval exchange transformations (IETs). In particular, they defined the notion of Roth type IETs and proved existence of the solution of cohomological equation for this class; they also showed that IET of Roth type form a full measure set in the parameter space of IETs.

In a very fresh joint work with Erwan Lanneau and Stefano Marmi we get a certain generalization of this result for linear involutions that can be considered as a natural extension of IETs to non-orientable case.

Bifurcations of families of vector fields on the two-dimensional sphere

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There exists pair of weakly topologically equivalent vector fields on the two-sphere with a parabolic cycle such that their generic one-parameter unfoldings are not equivalent ([1]). Classification of generic one-parameter unfoldings of vector fields with parabolic cycles is given. Any pair of weakly topologically equivalent degenerate vector fields of codimension 1 without parabolic cycle can have only equivalent one-parameter families as unfoldings.

An overview of bifurcations of generic one-parameter families and proof of the result stated above is provided.

The author is supported by RFBR project 16-01-00748-a

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Holomorphic normal form of nonlinear perturbations of nilpotent vector fields

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We consider germs of holomorphic vector fields at a fixed point having a nilpotent linear part at that point, in dimension $n \geq 3$. Based on Belitskii's work, we know that such a vector field is formally conjugate to a (formal) normal form. We give a condition on that normal form which ensure that the normalizing transformation is holomorphic at the fixed point. We shall show that this sufficient condition is a *nilpotent version* of Bruno's condition (A). In dimension 2, no condition is required since, according to Stróżyna-Żoładek, each such germ is holomorphically conjugate to a Takens normal form. Our proof is based on Newton's method and $\mathfrak{sl}_2(\mathbb{C})$-representations.

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On a heavy quantum particle

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We consider Schrödinger equation for a particle on a flat n-torus in a bounded potential, depending on time. Mass of the particle equals $1/\mu^2$, where μ is a small parameter. We show that the Sobolev H^ν-norms of the wave function grow approximately as t^ν on the time interval $t \in [-t_*, t_*]$, where t_* is slightly less than $O(1/\mu)$.

On classification of Morse-Smale dynamical systems with few non-wandering points

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We consider Morse-Smale flows and diffeomorphisms with the non-wandering set consisting of three fixed points (two nodes and a saddle). We study the topological structure of supporting manifolds. The question of the topological classification is considered also. The results obtained in collaboration with V. S. Medvedev.

This work was supported by Russian Scientific Found (RNF), project 17-11-01041.

Flat and hyperbolic enumerative geometry

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This talk (based on joint work with V. Delecroix, E. Goujard and P. Zograf) bridges certain aspects of flat and hyperbolic enumerative geometry.

We derive from the Kontsevich's count of metric ribbon graphs a formula for the Masur–Veech volume of the moduli space $\mathcal{Q}_{g,n}$ of quadratic differentials in terms of intersection numbers of ψ-classes.

We show that Mirzakhani's frequencies of simple closed hyperbolic geodesics of different combinatorial types coincide with the frequencies of the corresponding square-tiled surfaces.

Using our recent equidistribution results for square-tiled surfaces of fixed combinatorial type we derive applications for the count of meanders on surfaces of arbitrary genus.

We conclude with several conjectures concerning the shape of a typical multicurve on a surface of large genus and the shape of a typical square-tiled surface of large genus.

Posters

Energy function for 3-diffeomorphisms with one-dimensional surface attractor and repeller

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The Lyapunov function was introduced by A. M. Lyapunov to study the stability of equilibrium states of differential equations. A smooth Lyapunov function in which the set of critical points coincides with the chain-recurrent set of the system is called the energy function. For example, such functions always exist for flows, whereas examples of cascades are known which do not have an energy function. So there always exists a Morse-Lyapunov function for arbitrary gradient-like Morse-Smale diffeomorphisms, but, in general, there is no Morse energy function. The first example of such diffeomorphism was built on the 3-sphere by Picston in 1977 based on the wild arc of Artin-Fox. The Pikston construction allows generalization to any dimensions, thereby giving in the general case a negative answer to the question of the existence of an energy function in systems with regular dynamics. The Pikston construction

allows generalization to any dimensions, thereby giving in the general case a negative answer to the question of the existence of an energy function in systems with regular dynamics. The more surprising is the fact that there exists an energy function for cascades with chaotic dynamics, discovered in 2015 by V. Z. Grines, M. K. Barinova and O. V. Pochinka. The Pikston construction allows generalization to any dimensions, thereby giving in the general case a negative answer to the question of the existence of an energy function in systems with regular dynamics. The more surprising is the fact that there exists an energy function for cascades with chaotic dynamics, discovered in 2015 by V. Z. Grines, M. K. Barinova and O. V. Pochinka. They established the existence of the energy function for surface omega-stable diffeomorphisms with one-dimensional basis sets and for rough three-dimensional diffeomorphisms with surface basis sets. This report is a continuation of this topic. We consider Ω-stable 3-diffeomorphisms, the nonwandering set of which consists of exactly one surface attractor and one surface repeller. We build an example of such a diffeomorphism and prove the existence of the energy function of such diffeomorphisms.

Central Limit Theorem for Laguerre Ensembles Under Double Scaling

Sergey Berezin [1] and Alexander Bufetov [2]

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The Central Limit Theorem is obtained for additive functionals of Laguerre random matrices in a double-scaling limit as both the matrix size and the scaling parameter of the model go to infinity. The Riemann–Hilbert approach is used to find the asymptotic behaviour of the exponential moments.

Difference between smooth and formal classifications of Morse transitions of implicit ODE

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A topological classification of Morse transitions in families of implicit ordinary differential equations

$$F(x, y, y', \varepsilon) = 0$$

depending on a parameter $\varepsilon \in \mathbb{R}$ is obtained in [1], consists of four types, and does not contain continuous invariants. The phase portrait of one of them — a disk appearing from nothing — is shown below.

$\varepsilon < 0$ $\varepsilon = 0$ $\varepsilon > 0$

The shown transition has the topological normal form

$$y'^2 + x^2 + y^2 = \varepsilon.$$

Its asymptotic (formal) normal form

$$y'^2 + x^2 + y^2 + c(\varepsilon)y^3 = \varepsilon$$

with one functional invariant c^2 is found in [2].

It turns out that its smooth normal form is more complicated and contains at least one more invariant — a flat function of two variables which can not be killed by smooth changes of the parameter ε and the variables x, y depending on the parameter.

The work is partially supported by RFBR-16-01-00766.

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On the circumcenters of triangular orbits in elliptic billiard and complex reflexion law

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Here is a little problem: take an ellipse and look at the triangular billard orbits inscribed in it. What can we say about the locus of their circumcenters? We prove that it is an ellipse [Fie18]. The method of proof is based on a complex reflexion law introduced by Alexey Glutsyuk [Glu14b, Glu14a, Glu17] in order to study Ivrii's conjecture on periodic orbits together with its analogues for pseudo-billiards and complex billiards. Olga Romaskevich [Rom14] proved that the incenters of the triangular orbits form an ellipse, by using the above-mentioned complex reflection law. Our proof follows a similar scheme, as in [Rom14].

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Evolution of moments of isotropic Brownian stochastic flows

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We consider the following stochastic integral equation:

$$x(u,t) = u + \int_0^t \int_{\mathbb{R}} \varphi(x(u,s) - q)\, W(dq, ds), \quad t \geqslant 0, \quad u \in \mathbb{R},$$

where W is the Wiener sheet on $\mathbb{R} \times \mathbb{R}_+$ and the function $\varphi \in C_0^\infty(\mathbb{R})$ is non-negative and such that $\varphi(q) = \varphi(-q)$, $q \in \mathbb{R}$, and $\int_{\mathbb{R}} \varphi^2(q)\, dq = 1$. It is known (see [1]) that under these conditions on the function φ this equation has a unique strong solution for every $u \in \mathbb{R}$, and the corresponding family of random mappings $x(\cdot, t)\colon \mathbb{R} \to \mathbb{R}$, $t \geqslant 0$, defines an isotropic Brownian stochastic flow of C^∞-diffeomorphisms (see [3]).

For this stochastic flow we establish the precise asymptotic behaviour of all mixed moments and of all moments of the interparticle distance.

These results can be easily extended to the general case of one-dimensional isotropic Brownian stochastic flows with some non-restrictive conditions imposed on their covariance function.

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Topological conjugacy of Ω-stable flows without limit cycles on surfaces

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Recall that topological conjugacy for flows means existence of a homeomorphisms sending trajectories of one flow into trajectories of another preserving orientation and time of moving. For continuous systems (flows) conjugacy differs of *topological equivalence*, which does not obvious preserve time of moving.

In this work we speak about Ω-stable dynamical systems on closed surfaces. The non-wandering set of such systems consists of a finite number of hyperbolic periodic orbits and fixed points which does not construct cycles. Considered systems are not structural stable in a general case because of opportunity of a heteroclinic tangency. In case of flows such tangency means coincidence of two separatrices of different saddle points; such separatrix is called *connection*.

So, any neighbourhood of an Ω-stable system in the space of dynamical systems may consist infinitely many pairwise different classes of topological conjugacy. The parameters describing classes in some such neighbourhood are called *moduli of topological conjugacy*.

In 1978 J. Palis [1] invented a continuum of topologically non-conjugated dynamical systems in a neighbourhood of a system with a heteroclinic tangency on a surface. He described a neighbourhood of a heteroclinic tangency trajectory and showed that conjugacy needs coincidence of moduli. A modulus depends on eigenvalues of the saddles whose separatrices tangent.

Later W. de Melo and S. Van Strien in [2] in 1987 characterised a class of surface diffeomorphisms with a finite number of moduli. They found that a chain of saddles taking part in a heteroclinic tangency can not be more than two. But there is no need in such property in case of flows.

In this work we show that a number of moduli for flows without limit cycles is finite and coincides with a number of connections. A complete topological invariant for such systems is four-colour graph carrying information about dividing the phase space into domains with similar trajectories behaviour (cells). Edges of this graph are equipped with analytical parameters – moduli, connected with saddle connections. Such graph generilize the Oshemkov-Sharko three-colour graph [3] for Morse-Smale flows on surfacesand four-colour graph from [4], which is the invariant of topological *equivalence* of Ω-stable flows without limit cycles on surfaces.

Acknowledgements. Author thanks Olga Pochinka for irreplaceable help. The research was funded by RFBR according to the research project No 18-31-00022 mol_a.

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Classification of one-dimensional attractors by means of Pseudo-Anosov homeomorphisms

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In the present paper axiom A diffeomorphisms of a closed 2-manifold M^2 of genus $p \geq 2$ whose nonwandering sets contain spaciously situated basic set Λ are considered. It is shown that the problem of topological classification of such diffeomorphisms is closely related to the problem of topological classification of Pseudo-Anosov homeomorphisms on surfaces.

Definition 1. One-dimensional basic set Λ of an axiom A diffeomorphism $f\colon M^2 \to M^2$ is called if perfect its complement $M^2 \setminus \Lambda$ consists of a finite union of open disks.

In the paper [1] it was announced that for any Pseudo-Anosov homeomorphisms of a surface it is possible to construct a homotopic structurally stable diffeomorphism with a perfect

basic set by means of a surgery operation. However, the problem of topological classification of such diffeomorphisms was not considered there.

Theorem 1. *Let $f\colon M^2 \to M^2$ be a diffeomeorphism of an orientable 2-manifold M^2 of genus $p \geq 2$ with a perfect spaciously situated basic set Λ, then there exist a continuous map $h\colon M^2 \to M^2$ homotopic to the identity and a Pseudo-Anosov homeomorphism $PA_f\colon M^2 \to M^2$ such that the following diagram commutes $h \circ f = PA_f \circ h$.*

We say that a periodic point point p is tagged if the set $h^{-1}(p) \cap \Lambda$ consist of exactly two distinct points. Let B denote the set of all tagged points. The main result is the following theorem.

Theorem 2. *Let f and f' be A-diffeomorphisms of a closed orientable 2-manifold M^2 of genus $p \geq 2$, with perfect spaciously situated attractors Λ and Λ' correspondingly. Then there exists a homeomorphism $g\colon M^2 \to M^2$ such that the following diagram commutes $f'|_{\Lambda'} = g \circ f \circ g^{-1}|_{\Lambda'}$ if and only if there exists a homeomorphism $h\colon M^2 \to M^2$ such that $P_{f'} = h \circ P_f \circ h^{-1}$ and $h(B) = B'$.*

Acknowledgments. Research is done with financial support of Russian Science Foundation (project 17-11-01041)

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The space of cubic Newton maps with persistent parabolics

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We study the space of cubic Newton maps for entire maps that take the form $(z^2 + c)e^z$ and the space is parametrized by a single complex parameter $c \neq 0$. These Newton maps are rational, $z \mapsto f_c(z) = z - \frac{z^2+c}{z^2+2z+c}$, with a persistent parabolic fixed point at ∞ and two superattracting fixed points in $\mathbb{C}$, and all have a connected Julia set [M1]. We show that stable components, the components of cubic Newton maps for which the free critical point belongs to attracting basins and the basin of the parabolic fixed point at ∞, are topological disks. When the free critical point belongs to the basin of ∞, we consider quasiconformal conjugacy classes in a corresponding stable component, which is a topological disk comprised of half-closed strips, which is a bi-infinite cylinder obtained explicitly as a class of quasiconformal deformations of some f_c in this space [BMMS]. Each deformation class contains a unique such f_c, which is conformally rigid and we call these maps PostCritically Minimal and PostCritically Non-Minimal according to the number of iterates its free critical point takes to reach the other non-free critical point in the parabolic basin [M2]. Furthermore, by means of quasiconformal surgery we construct a canonical bijection from the hyperbolic components of the space of standard cubic Newton maps of polynomials to the stable components of this one-parameter space [M1, M2].

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Modulus of continuity away from zero for correlation measures in substitution tiling systems

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A historically powerful tool in ergodic theory is the spectral analysis of Koopman operators arising from a dynamical system. The spectral theorem identify these operators with the so called spectral measures, making them an important object of study. As an example of this, you can read the excellent book of Queffelec [Que09], where the author characterizes some spectral measures in the case of one-dimensional symbolic substitutions, as a combination of Dirac's deltas.

More recently, Bufetov and Solomyak studied these measures in tilings systems of the real line arising from substitu-

tions [BS14], but with the difference of being focused in continuous spectra, i.e., the weakly-mixing case. They were inspired by some previous works of Hof [Hof97]. Riesz products are used to characterize spectral measures, and some methods from Diophantine approximation theory to estimate these products.

Some of those results are generalized to some Cantor systems in a more recent paper of the same authors. In other direction, one result of [BS14] is generalized to substitution tiling systems of $\mathbb{R}^d$: in [Emm17] the modulus of continuity of spectral measures around zero is characterized by some Radon measure. In this way, the aim of the actual paper is the study of decay rates of the spectral measures of substitution tiling systems of $\mathbb{R}^d$, but at points different from zero. The next theorem is the main result of this paper and is a natural generalization of Theorem 5.1 in [BS14]:

Theorem 1. *Let ζ a tile-substitution with expansion L over the alphabet $A = \{T_1, \dots, T_m\}$ and $f = \mathbf{1}_{\mathfrak{X}_{T_i}}$ (characteristic function of the set of tilings covering $0 \in \mathbb{R}^d$ with the tile T_i). Suppose L is diagonalizable over $\mathbb{C}$, its eigenvalues are all Galois conjugates and there exists one conjugate outside the unit disk and not in the spectrum of L. Consider the set of Ξ the set of ω's such that $v \notin \langle\{\omega\}\rangle^\perp$ and $\liminf_{l\geq 0} |\omega \cdot L^l v| > 1/2$, for some $v \in \mathbb{R}^d$ depending on ζ. Then, for $\varepsilon > 0$ there exist constants C_ε, $\gamma > 0$ such that*

$$\sigma_f([\omega_1 - r, \omega_1 + r] \times \cdots \times [\omega_d - r, \omega_d + r]) \leq C_\varepsilon \log(1/r)^{-\gamma}, \quad r \searrow 0.$$

for ω's in Ξ such that $|\sum_{i=1}^{m_1} \eta_1^i \omega \cdot u_1^i| \in [\varepsilon, 1/\varepsilon]$, where $\{u_1^i\}_{i=1}^{m_1}$ are the eigenvectors of L associated to the greater (in modulus) eigenvalue and m_1 its multiplicity.

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Invariant surfaces for Toric type foliations in dimension 3

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Few years ago, René Thom asked the following question: *Is there always an invariant surface for a given codimension one foliation on* $(\mathbb{C}^n, 0)$*?* C. Camacho and P. Sad proved, in 1982, that any foliation of $(\mathbb{C}^2, 0)$ has an invariant curve. For greater dimension, there are results of Cano-Cerveau and Cano-Mattei, that give a positive answer to Thom's question for non-dicritical foliations. However, when the existence of dicritical components of the exceptional divisor is allowed, Jouanolou gives a collection of conic foliations on $(\mathbb{C}^3, 0)$ without invariant surface.

What we present in this poster, is a result of existence of invariant surface for, dicritical or not, toric type foliations on $(\mathbb{C}^3, 0)$. By toric type we mean the existence of a combinatorial

process of reduction of singularities. This is a slightly stronger condition than being a Newton non-degenerate foliation (in the classical sense in terms of Newton polyhedra). In order to prove this three-dimensional result of local nature, we state and prove the following global result in dimension two: "The isolated invariant curves of toric type foliations on compact toric surfaces are closed curves".

On displacement of viscous liquid in a quasi 1D system

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An explicitly solvable quasi 1D model of oil displacement is studied. The problem of recovering of the reservoir geometry is solved by means of a fixed point algorithm. The stability of solution is studied in various functional classes.

On Number Rigidity for Pfaffian point processes

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Our first result states that the orthogonal and symplectic Bessel processes are rigid in the sense of Ghosh and Peres. Our argument in the Bessel case proceeds by an estimate of the variance of additive statistics in the spirit of Ghosh and Peres. Second, a sufficient condition for number rigidity of stationary Pfaffian processes, relying on the Kolmogorov criterion for interpolation of stationary processes and applicable, in particular, to pfaffian sine-processes, is given in terms of the asymptotics of the spectral measure for additive statistics.

Joint work with A. Bufetov and Y. Qiu.

On periodic data of diffeomorphisms with one saddle orbit

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In the study of discrete dynamical systems, i.e., the study of orbits of self-mapsf defined on a given compact manifold, the periodic behavior plays an important role. In the last forty years there was a growing number of results showing that certain simple hypotheses force qualitative and quantitative properties (like the set of periods) of a system. One of the best-known results is the title of the paper "Period three implies

chaos for the interval continuous self-maps" [5]. The effect described there was discovered by O. M. Šarkovs'kiı in [7]. The most useful tools for proving the existence of fixed points or, more generally, of periodic points for a continuous self-map f of a compact manifold is the Lefschetz fixed point theorem and its improvements (see, for instance [2] and [3]). The Lefschetz zeta-function simplifies the study of the periodic points of f. This is a generating function for all the Lefschetz numbers of all iterates of f. The periodic data of diffeomorphisms with regular dynamics on surfaces were studied by means zeta-function in a series of already classical works by such authors as Paul R. Blanchard, John M. Franks, Rufus Bowen, Steve Batterson, John Smillie, William H. Jaco, Peter B. Shalen, Carolyn C. Narasimhan, and others. A description of periodic data of gradient-like diffeomorphisms of surfaces was given in [1] by means of classification of periodic surface transformations obtained by Jakob Nielsen [6]. In [4], the authors show that the study of periodic data of arbitrary Morse–Smale diffeomorphisms on surfaces is reduced by filtration to the problem of computing periodic data of diffeomorphisms with a unique saddle periodic orbit. Polar diffeomorphisms of the surface are considered, that is, iffeomorphisms having a single sink and a single source periodic orbit. A classic example of such a diffeomorphism is the sink-source diffeomorphism, which has no saddle points and exists only on a two-dimensional sphere. However, the addition of even a single saddle orbit greatly expands the class of polar diffeomorphisms on surfaces. All possible types of periodic data for such polar diffeomorphisms have been established, and it is shown that the saddle orbit always has a negative orientation type. It is proved that ev-

ery orientable surface admits a Morse–Smale diffeomorphism with one saddle orbit that preserves orientation. It is shown that these diffeomorphisms have exactly three nodal orbits. In addition, all possible types of periodic data for such diffeomorphisms are established.

Acknowledgements. Author thanks Olga Pochinka for irreplaceable help. The research of periodic data for diffeomorphisms with a single saddle orbit was carried out with the support of grant RNF 17-11-01041.

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Normal forms of infinite index transversal systems with one consrtraint

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We consider locally analytic constrained systems with one constraint of the form

$$\begin{aligned} 0 &= F(x,u,y) \\ \dot{u} &= v_1(x,u,y), \quad (x,u,y) \in (\mathbb{C}\times\mathbb{C}\times\mathbb{C}^n,0) \\ \dot{y} &= v_2(x,u,y) \end{aligned} \tag{1}$$

such that $F(0)=0$, $F'(0)\neq 0$, and the vector field $v=(v_1,v_2)$ does not vanish at the origin.

Systems of the form (1) of finite index, that is, the case when at least one partial derivative $\frac{\partial^k F}{\partial x^k}(0)$ does not vanish, were earlier considered in [1]. We say that the index of (1) is infinite if $\frac{\partial^k F}{\partial x^k}(0)$ for all $k\geq 0$. In this case, a normal form of (1) can be obtained from the following theorem, which is our main result:

Theorem 1. *A generic system* (1) *of infinite index is locally analytically equivalent to the system*

$$0 = u + y_1 x + y_2 x^2 + \ldots + y_n x^n + x^{n+1} A(x, {}'y), \dot{u} = 1, \dot{y} = 0,$$

where $'y = (y_2,\ldots,y_n)$, $A(x,0)\equiv 0$ *for* $n\geq 2$, *and* $A\equiv 0$ *for* $n=1$.

The proof of Theorem 1 is based on the results of [2, 3].

The research was supported by RFBR 16-01-00766.

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Periodic solutions of one-dimensional parabolic problems with hysteresis on the boundary

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We study problems which one can interpret as a temperature regulation in a domain via heating (and cooling) elements on the boundary. The regime of the heating elements on the boundary is based on the registration of thermal sensors inside the domain and obeys a hysteresis law. Long-time behaviour of such systems, in particular, existence and stability of periodic solutions, was studied in [1], [2] and [3] by Fourier method and Poincare map contraction property. It was established that the biggest difficulty in study of such systems occurs whenever the gap between the switching temperatures of the hysteresis operator $\beta - \alpha$ is too small. For example, in [3] coexistence of several periodic solutions with different stability properties is proved to be possible when $\beta - \alpha$ is small

enough. We considered one-dimensional system with the mean temperature values $u(0,t)$, $u(\frac{\pi}{2})$ and $u(\pi,t)$ (temperature values at the beginning, the center and 2 the end of the segment respectively) and proved that, in contrast to multi- dimensional case, for any $\beta - \alpha$ existence, uniqueness and stability of periodic solutions take place.

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Yu. S. Ilyashenko – from local to global

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We discuss some of Yu. S. Ilyashenko results and the individuality of his mathematical style.

Dynamics of geodesics of meromorphic connections on compact Riemann surfaces

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In this work we have studied relation among three objects: quadratic differentials ($\mathcal{QD}$), meromorphic flat surfaces ($\mathcal{MFS}$) and meromorphic connections ($\mathcal{MC}$) on Riemann surfaces. First of all, we have shown that there exists a subset $\mathcal{A} \subsetneq \mathcal{MC}$ such that $\mathcal{QD} \cong \mathcal{A}$. We have shown that if (S, g) is a meromorphic flat surface (S is a Riemann surface with punctures and g is a flat metric outside the punctures) then there exists a meromorphic connection such that g will be the metric adapted to the meromorphic connection. In particular, $\mathcal{MFS} \subsetneq \mathcal{MC}$.

In this work we have defined a notion of argument for a meromorphic connection and by using it we have proved some properties of self-intersecting geodesics. As a consequence, we have seen that the imaginary part of residues of poles of a meromorphic connection does not significantly affect dynamical properties of the meromorphic connection.

We also studied local behavior of geodesics of a meromorphic connection in a neighbourhood of Fuchsian poles. We have seen that if the omega-limit set of a geodesic of a meromorphic connection contains a pole with real part of residue less than or equal to -1 then the omega-limit set is exactly the pole. Consequently, there is not a geodesic which is dense in S.

Strict analytic classification of simplest germs of semi-hyperbolic maps

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A germ of a biholomorphic map $F : (\mathbb{C}^2, 0) \to (\mathbb{C}^2, 0)$ is called *semi-hyperbolic* if one of its multipliers is equal to one and the second one is hyperbolic. For example, the 1-time shift $F_\lambda = g^1_{v_\lambda}$ along the vector field $v_\lambda = x^2 \frac{\partial}{\partial x} + \lambda y \frac{\partial}{\partial y}$ $(\lambda \in \mathbb{R}_+)$ is semi-hyperbolic.

Two semi-hyperbolic germs will be called *strictly* equivalent if the local change of coordinates conjugating them has the form $(x + o(x^2), y + o(x))$, $x \to 0$. Let $\mathbf{F}_\lambda$ be the class of semi-hyperbolic germs strictly formally equivalent to the germ F_λ. We will show, that strict analytic classification of $\mathbf{F}_\lambda$ has functional modules.

Consider the class M consisting of collections $(A_\pm, B_\pm, A, B, C)$ such that: $C \in \mathbb{C}$; A_+, B_+ are holomorphic in $(\mathbb{C}, \infty) \times (\mathbb{C}, 0)$ and $A_+(t, \tau) = O(1)$, $B_+(t, \tau) = O(t^{-1})$, $t \to \infty$; A, B are holomorphic in $(\mathbb{C}, 0)$ and $A(\tau) = O(\tau)$, $B(\tau) = O(\tau^2)$, $\tau \to 0$; A_-, B_- are holomorphic in $(\mathbb{C}^2, 0) A_-(t, \tau) = O(t^2)$, $B_-(t, \tau) = O(t)$, $t \to 0$.

Theorem 1. *The space M is the space of functional modules of strict analytic classification of germs of $\mathbf{F}_\lambda$.*

Theorem 2. *A germ $F \in \mathbf{F}_\lambda$ has a holomorphic central manifold if and only if $C = B_\pm(t, 0) = 0$ for corresponding components of its module.*

Denote by $\mathbf{V}_\lambda$ the class of strict formally equivalency of the v_λ.

Theorem 3. *A germ $F \in \mathbf{F}_\lambda$ is embaddable into a flow (i.e., it is a 1-time shift along a field $v \in \mathbf{V}_\lambda$) if and only if $A_\pm = B_\pm = 0$ for corresponding components of its module. In this case, the components C, A are the Martinet-Ramis modules of the field v[1], and B is the Meshcheryakova-Teyssier module[2].*

Theorem 4. *Suppose that $F \in \mathbf{F}_\lambda$ has a holomorphic central manifold (then restriction f of the germ F to the central manifold is a parabolic map). Then the non trivial restrictions of the components of the module of the germ F to the line $\{\tau = 0\}$ are the Ecalle-Voronin modules of the germ f.*

The work is supported by a grant RFBR 17-01-00739 A

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Milnorian and non-Milnorian representations

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In 1977, Milnor formulated the following conjecture: every discrete group of affine transformations acting properly on the affine space is virtually solvable. We now know that this statement is false; the current goal is to gain a better understanding of the counterexamples to this conjecture. Every group that violates this conjecture "lives" in a certain algebraic affine group, which can be specified by giving a linear group and a representation thereof. Representations that give rise to counterexamples are said to be non-Milnorian. We will present the progress made so far towards classification of these non-Milnorian representations.

Various problems in parabolic equations with hysteresis

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We study several problems with discontinuous hysteresis, often called non-ideal relay. The main features of systems with hysteresis are the dependence of the output on the prehistory of the input, rate independence, and nonsmoothness. Hysteresis operators are applied in mathematical descriptions of

various physical, chemical and biological processes: thermocontrol, chemical reactors, ferro-magnetism, self-organisation and others.

We study periodic solutions in thermocontrol systems and pattern formation mechanisms for spatially distributed hysteresis, which describes biological systems. We explore a new mechanism for pattern formation, which we called "rattling". It seems the essence of the phenomena is related to the evolution of the Young measures.

The Weotzenböck theorem about invariants

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We propose a new and constructive proof of the old theorem of Weitzenböck that the ring of polynomial invariants of a linear vector field in $\mathbb{C}^m$ is finitely generated.

Finitely additive measures on the invariant foliations of Anosov diffeomorphisms

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Let $F : M \to M$ be a C^{2+s} topologically mixing Anosov diffeomorphism of a compact Riemannian manifold whose stable and unstable foliations are oriented and have dimensions $d-k$ and k. We construct a Banach space $\mathfrak{B}$ of currents of degree k whose elements induce finitely additive measures on 'nice' subsets of k-dimensional admissible leaves, whose tangent space everywhere belongs to the unstable cone of the map F. All measures induced by the elements of $\mathfrak{B}$ are regular in the following sense: given a C^{2+s} family X_t of subsets of admissible leaves with C^1 boundary, the measure of X_t is a C^1 function in t.

The constructed Banach space $\mathfrak{B}$ contains the unique σ-additive positive measure μ_U on the unstable foliation such that $F^*\mu_U = e^h\mu_U$, where h is topological entropy of F. Since any C^{1+s} smooth function $f : M \to \mathbb{R}$ is a bounded multiplier in $\mathfrak{B}$, we can consider the measure $f\,d\mu_U$ as an element of $\mathfrak{B}$. The integration of elements of $\mathfrak{B}$ on nice subsets defines a continuous linear functional on $\mathfrak{B}$, which allows to deduce the following effective equidistribution result.

Theorem 1. *If a C^{2+s} topologically mixing Anosov diffeomorphism $F : M \to M$ has oriented foliations and satisfies certain bounded distortion estimates, then there exists $\zeta \in (0, 1)$ and a*

constant $C > 0$ such that, for any $f \in C^{1+s}(M)$ and any unit unstable ball $B = B(x)$ centered at $x \in M$, we have

$$\left| \frac{1}{\mu_U(F^n B)} \int\limits_{F^n B} f \, d\mu_U - \int_M f \, d\mathbf{m} \right| \leq C \cdot \zeta^n \cdot \|f\|_{C^{1+s}(M)},$$

where $\mathbf{m}$ is the measure of maximal entropy of the map F normalized so that $\mathbf{m}(M) = 1$.

The key argument is the fact that the pullback operator F^* is quasi-compact in $\mathfrak{B}$ and has spectral radius e^h. Furthermore, we show that the Banach space $\mathfrak{B}$ admits a finite dimensional subspace, spanned by the eigenvalues which are very close to e^h, whose elements induce regular finitely additive measures, which are invariant under the holonomy in the stable direction.

Acknowledgements. The work was supported by the Academic Fund Program of the National Research University Higher School of Economics (HSE) in 2018-2019 (grant number 18-05-0019) and by the Russian Acamedic Excellence Project '5-100'. The author is a Young Russian Mathematics award winner and would like to thank its sponsors and jury.